

# Dynamics as a probe for population imbalanced Fermionic Systems

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We investigate a population-imbalanced two-species fermionic system where the resonantly-paired fermions combine to form bosonic molecules via Feshbach interaction. The natural dynamics of the system is studied and it is shown that the oscillation of the condensate fraction is periodic or quasi periodic, depending on the value of Feshbach coupling. We describe how a time dependent magnetic field can be used to study the natural frequencies and thus explore the momentum space structure of the population imbalanced system.

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## I. INTRODUCTION

Ultracold fermionic atoms have attracted a lot of attention in the last decade. The fermions here can exhibit BCS pairing or can form molecules and undergo Bose Einstein condensation(BEC). The crossover from BCS to BEC can be controlled by a magnetic field capable of providing Feshbach resonance[1–17]. An interesting variation in this situation is the introduction of an imbalance in the fermion number. Pairing now has to work around the fact that not all fermions of type A(one of the species) have a fermion of type B to pair with. This is equivalent to considering superconductivity in the presence of an external magnetic field which creates an imbalance of spin up and spin down states. The consequence can be exotic pairing states like FFLO[18, 19], Sarma (breached pair) phase[20–23] , phase separation [16, 24]. These are, however, notoriously difficult to detect experimentally.

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Dynamics is a very important attribute of ultracold systems and has been studied extensively. R.A. Barankov et al. discussed the collective nonlinear evolution of BCS state when the pairing interaction is turned on instantaneously [25]. V. A Andreev et al. showed that when the position of the Feshbach resonance is changed abruptly, the superfluid undergoes a coherent BEC-to-BCS oscillation[26]. M. H. Szyman'ska et al.[27] addressed the situation when the Feshbach magnetic field jumps suddenly, and studied the short time dynamics that followed – damped oscillations with an amplitude depending on initial conditions. They also argued that atom-molecule oscillations are negligible for all practical purposes. All these authors concentrated on following the system after a rapid change. This entails the study of the nonlinear term in the evolution equation.

In this paper, we aim at studying the dynamical properties of a population-imbalanced Fermi gas but without introducing a sudden change in Feshbach magnetic field. We focus on the linear dynamics of the system when a natural fluctuation of BEC condensate occurs along the BCS-BEC crossover path, and the system tries to relax back to the steady state. In the process we find that it shows periodic or quasiperiodic oscillations. The strength of the Feshbach term determines whether the oscillation will be periodic or quasi periodic. The frequencies of motion are found to be sensitive to the pairing in the momentum space. We propose that a time dependent Feshbach coupling can be used to probe the pairing structures in momentum space. This can provide another handle on exploring experimentally the exotic pairing states.

## I. MODEL HAMILTONIAN

Here we start with a two-species fermionic system. In addition to the fermion-fermion interaction ( denoted by  $g_1$ ), there is an additional interaction ( $g_2$ ) of the Feshbach variety which couples two fermions to form a bosonic molecule . Our model resembles the one used in [10, 11, 28] .  $\psi_{p\uparrow}$  and  $\psi_{-p\downarrow}$  denote the fields for the two fermions, and  $\phi$  represents the Bosonic field.

Our Hamiltonian is :

$$H = \sum_p \epsilon_p (\psi_{p\uparrow}^\dagger \psi_{p\uparrow} + \psi_{-p\downarrow}^\dagger \psi_{-p\downarrow}) + g \sum_p \psi_{p\uparrow}^\dagger \psi_{-p\downarrow}^\dagger \psi_{-p\downarrow} \psi_{p\uparrow} + g_2 \sum_p (\psi_{p\uparrow} \psi_{-p\downarrow} \phi^\dagger + \psi_{p\uparrow}^\dagger \psi_{-p\downarrow}^\dagger \phi) + \epsilon_b \phi^\dagger \phi \quad (1)$$

Next we calculate the commutation relations in order to arrive at the equations of motion. This is in line with the Ehrenfest Theorem, which relates the time derivative of the expectation value for a quantum mechanical operator to the commutator of that operator with the Hamiltonian of the system.

Heisenberg operators are introduced as  $\psi_{H\uparrow}(pt) = e^{iHt/\hbar} \psi_{p\uparrow} e^{-iHt/\hbar}$  and  $\psi_{H\downarrow}^\dagger(pt) = e^{iHt/\hbar} \psi_{p\downarrow}^\dagger e^{-iHt/\hbar}$ . The equations of motion are :

$$\begin{aligned} i\hbar \frac{\partial \psi_{H\uparrow}}{\partial t} &= e^{iHt/\hbar} [\psi_{p\uparrow}, H] e^{-iHt/\hbar} = \epsilon_p \psi_{H\uparrow} + g \langle \psi_{p\uparrow} \psi_{p\downarrow} \rangle \psi_{H\downarrow}^\dagger + g_2 \psi_{H\downarrow}^\dagger \phi \\ i\hbar \frac{\partial \psi_{H\downarrow}^\dagger}{\partial t} &= e^{iHt/\hbar} [\psi_{p\downarrow}^\dagger, H] e^{-iHt/\hbar} = \epsilon_p \psi_{H\downarrow}^\dagger + g \langle \psi_{p\downarrow}^\dagger \psi_{p\uparrow}^\dagger \rangle \psi_{H\uparrow} + g_2 \psi_{H\uparrow} \phi^\dagger \end{aligned} \quad (2)$$

Let us now define the pair wavefunction.

$$O_p = \langle \psi_{H\uparrow} \psi_{H\downarrow} \rangle \quad (3)$$

It turns out that

$$i\hbar \frac{\partial O_p}{\partial t} = 2\epsilon_p \langle \psi_{H\uparrow} \psi_{H\downarrow} \rangle + g \langle \psi_{\uparrow} \psi_{\downarrow} \rangle (N_p - 1) + g_2 \phi (N_p - 1) \quad (4)$$

Here  $N_p = N_1 + N_2$ , i.e, the total population corresponding to a particular momentum  $p$ . Now,  $\langle \psi_{H\uparrow} \psi_{H\downarrow} \rangle$  represents the expectation value of the pair wave function in the Heisenberg picture while  $\langle \psi_{\uparrow} \psi_{\downarrow} \rangle$  is the same quantity in the Schroedinger's picture. Since these are expectation values, they should not depend on the choice of the representation. So we can replace  $\langle \psi_{H\uparrow} \psi_{H\downarrow} \rangle$  by  $O_p$  as well. Moreover, when both the pairing states are occupied, (e.g, in region of BCS pairing),  $N_p = 2$  Therefore,

$$i\hbar \frac{\partial O_p}{\partial t} = (2\epsilon_p + g) O_p + g_2 \phi \quad (5)$$

As for the evolution of  $\phi$

$$i\hbar \frac{\partial \phi}{\partial t} = g_2 \sum_p O_p + \epsilon_b \phi \quad (6)$$

A quick comparison shows that these are linearized versions of the evolution equations in [26, 27], which is the relevant part for our calculation. The non linear terms are essential for dynamics following a sudden quench.

## II. SYSTEM IN EQUILIBRIUM:

The first test of the new theory is that, whether we can reproduce previously established results as special cases of the new formalism. So we try to go back to the static case using these dynamical equations.

We define a single particle Green's function in the momentum space.

$\mathcal{G}(pt, p't') = -\langle [\psi_{H\uparrow}(pt)\psi_{H\uparrow}^\dagger(p't')] \rangle$  and  $\mathcal{F}^\dagger(pt, p't') = -\langle [\psi_{H\uparrow}^\dagger(pt)\psi_{H\uparrow}(p't')] \rangle$  and find that

$$i\hbar \frac{\partial}{\partial t} \mathcal{G}(pt, p't') = i\hbar(p - p')\delta(t - t') + \epsilon_p \mathcal{G}(pt, p't') + g_{eff} \langle \psi_{p\uparrow} \psi_{p\downarrow} \rangle \mathcal{F}^\dagger(pt, p't') \quad (7)$$

A direct analogy with the standard BCS [29] leads to the relation

$$g_{eff} \sum_p \langle \psi_{p\uparrow} \psi_{p\downarrow} \rangle = g \sum_p \langle \psi_{p\uparrow} \psi_{p\downarrow} \rangle + g_2 \phi \quad (8)$$

In the static case, let the condensate order parameter be a constant, i.e.,  $\frac{\partial \phi}{\partial t} = 0$

From Equations (6) and (8),  $g_{eff} = g - \frac{g_2^2}{\epsilon_b}$ . If the fermion-fermion four point interaction is an attractive one (which is indeed the case for a BCS superfluid), then taking The effective interection  $g_{eff}$  in the attractive sense and writing  $g = -|g_1|$ , we arrive at

$$g_{eff} = g_1 + \frac{g_2^2}{\epsilon_b} \quad (9)$$

This expression matches with the one obtained using diagrammatic methods [11] and variational technique [28] So, here we are able to reproduce the same results without having to use a trial form of the ground state of the system. The Ehrenfest theorem and the commutation relations have directly landed us here, and this treatment is certainly more general in nature.

### III. SYSTEM OUT OF EQUILIBRIUM : FREQUENCIES OF OSCILLATION

Let,  $\tilde{O}_p$  be the fluctuation in  $O_p$ , and  $\tilde{\phi}$  be the fluctuation in  $\phi$ , Therefore,

$$i\hbar \frac{\partial \tilde{O}_p}{\partial t} = (2\epsilon_p + g)\tilde{O}_p + g_2\tilde{\phi} \quad (10)$$

$$i\hbar \frac{\partial \tilde{\phi}}{\partial t} = g_2 \sum_k \tilde{O}_p + \epsilon_b \tilde{\phi} \quad (11)$$

We take the respective Fourier transforms and find that  $O_p(\omega) = -\frac{g_2\phi(\omega)}{a_p + \hbar\omega}$

So far, the treatment has been general, and the population balance or imbalance has never been taken into account. Now we come to the specific situation where there are two species of Fermions, or may be, two hyperfine states of the same atom (for simplicity, we have used the  $\uparrow$  and  $\downarrow$  to denote them): but the one has a larger population than the other. In the momentum picture, this corresponds to a particular geometry. The natural choice is that of a two-tier structure, one consisting of the paired superfluid, and another comprising of a normal Fermi liquid made of the remaining unpaired fermions. It is shown that[28] a population-imbalanced Breached Pair state is stable only when the core is normal and the outer shell, superfluid.

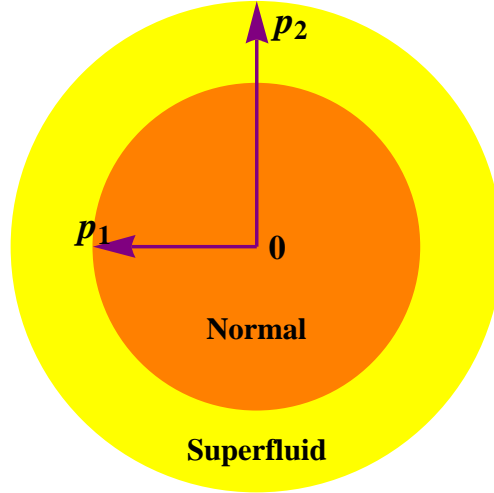


FIG. 1: Stable Structure for Population Imbalanced ‘Breached Pair’ State

In the above figure, the shell structure in momentum space is depicted. The unpaired majority fermions stay in the core region, which is a normal fluid. The pair superfluid forms

the outer shell, from momenta  $p_1$  to  $p_2$ , where  $p_1$  and  $p_2$  form the momenta boundaries between which the fermions form BCS-like pairs.

Experimental results obtained by the Ketterle group at MIT[30], too, confirms this structure. In a strongly interacting Fermi gas with imbalanced populations, they observed a superfluid region surrounded by a normal gas in the form of a shell structure in the coordinate space. Now, the structure we described above (normal core and superfluid shell), when mapped into real space via a Fourier Transformation shows a high density of superfluid in the centre ( i.e, the picture obtained by Shin et al.) provided the population imbalance is not too high. Our Fourier space calculation shown that this holds till  $p_2 > 1.26p_1$ . Now,  $\frac{p_1}{p_2}$  is the measure of the population imbalance of the system, and indeed from the experimental results obtained by Shin et al., this core and shell structure survives upto a population imbalance of 75 percent (i.e, when  $p_2 > 1.05p_1$ ). So our calculation matches with their findings within an error margin of 20 percent.

Thus,  $O_p(\omega)$  has to be summed over the superfluid region, i.e, over all  $p$  from  $p_1$  to  $p_2$ .

$$\sum_p O_p(\omega) = -g_2\phi \int_{p_1}^{p_2} \frac{p^2 dp}{(a_p + \hbar\omega)} \quad (12)$$

Let  $p_F$  be the Fermi momentum and  $M$  the mass of the atoms. Therefore  $a_p + \omega\hbar = ap + b$ , where  $a = \frac{2p_F}{M}$ ,  $b = -\frac{2p_F^2}{M} + g + \omega\hbar$ . Integrating Eq.(12)by parts, we obtain  $\sum_p O_p(\omega) = -g_2\phi(\omega)(f(p_2) - f(p_1))$  where  $f(a) = \frac{1}{a}[\frac{(p+\frac{b}{a})^2}{2} - \frac{b^2}{a^2}\ln(p + \frac{b}{a}) - \frac{2b}{a}(p + \frac{b}{a}) + \frac{2b^2}{a^2}\ln(p + \frac{b}{a})]$  Putting back in Eq(11), we obtain

$$\phi(\omega)[\epsilon_b + \hbar\omega - g_2^2(f(p_2) - f(p_1))] = 0$$

Which means,  $\phi(\omega)$  is zero if  $[\epsilon_b + \hbar\omega - g_2^2(f(p_2) - f(p_1))] \neq 0$ . Therefore, in the expansion of  $\phi(t)$ , only those  $\phi(\omega)$ s will survive for which

$$\epsilon_b + \hbar\omega - g_2^2(f(p_2) - f(p_1)) = 0 \quad (13)$$

Therefore,  $\phi(t) = \phi_1 e^{i\omega_1 t} + \phi_2 e^{i\omega_2 t} + \dots$

Here  $\omega_1, \omega_2 \dots$  are the solutions of equation (13).

To find out wheter there exists real values for  $\omega$ , we take resort to graphical solutions. We are only interested in real  $\omega$ , because that would give us solutions in the form of  $\phi(t) = \phi e^{i\omega t}$ ,

which denotes oscillation. If, on the other hand, we get imaginary solutions for  $\omega$ , then the solutions are of the form  $\phi(t) = \phi e^{\omega t}$  or  $\phi(t) = \phi e^{-i\omega t}$ . The first one signifies an exponential growth in  $\phi$  and is, therefore, unphysical. The second one marks exponential decay, and its effect should be negligible as time increases. So we look out for real solutions only.

From equation (13), the roots of  $\omega$  are given by  $\omega = f1(\omega)$ , where  $f1(\omega) = g_2^2(f(p_2) - f(p_1)) - \epsilon_b$ . So we plot  $\omega$  along X-axis and both  $f1(\omega)$  and  $\omega$  along Y-axis. The blue lines correspond to  $f1(\omega)$  and the red lines mark  $\omega$ . If the two lines cut at any point, we get real solutions of the equation at that point.

We scale all energies by the Fermi energy  $E_F$ , and all momenta by the Fermi momentum  $p_F$  of the majority species. Therefore, in this convention, mass of each particle gets fixed at 0.5.  $\hbar$  is taken as 1.

**When the Imbalance is Fixed and  $g_2$  is Varied:** The BCS pairing takes place near the Fermi surface, within a cut-off region. For standard superconductors, this cut-off is  $\hbar\omega_D$ ,  $\omega_D$  being the Debye frequency. For Ultracold atoms, the cut-off is  $\frac{4}{e^2}E_F = 0.541E_F$  [31]. Since we have scaled all energies by  $E_F$ , Fermi level corresponds to 1, and the lower cut-off is at  $1 - 0.541 = 0.459$ . Thus we have to choose  $p_1$  to lie between 0.459 and 1. We take  $p_2$  to be 1, the Fermi momentum.

We take  $p_1 = 0.5$ ,  $p_2 = 1$ ,  $\epsilon_b$  at 0.03. From figure 5.2, we see that when  $g_2$  is 0.1, there is only one point where  $f1(\omega)$  and  $\omega$  cut one another, i.e, only one real solution for  $\omega$ . A single solution exists for  $g_2 = 1$  as well. As this coupling is increased slightly, at  $g_2 = 1.7$ , there appears two such points, i.e, two real  $\omega$ s. Then, as  $g_2$  obtains higher values, there are always two real solutions for  $\omega$ .

So we can call  $g_2 = 1.7$  a critical coupling. If  $g_2$  is less than this coupling value, there is only one real frequency of oscillation in  $\tilde{\phi}(t)$ . Beyond it, there are two frequencies.

**When  $g_2$  is Fixed and the population imbalance is Varied:** Now  $p_2$  has been fixed at 1,  $\epsilon_b$  at 0.03. We vary  $p_1$ , which measures the amount of imbalance, because it is up to momentum  $p_1$  that the majority fermions remain unpaired.

In figure 5.3, we observe that when the population imbalance is very low ( $p_1$  is 0.5, i.e, a value slightly higher than the lower cut-off for pairing), the double frequency region appears at  $g_2 = 1.7$ . As the imbalance is increased gradually, the value of the critical coupling decreases.

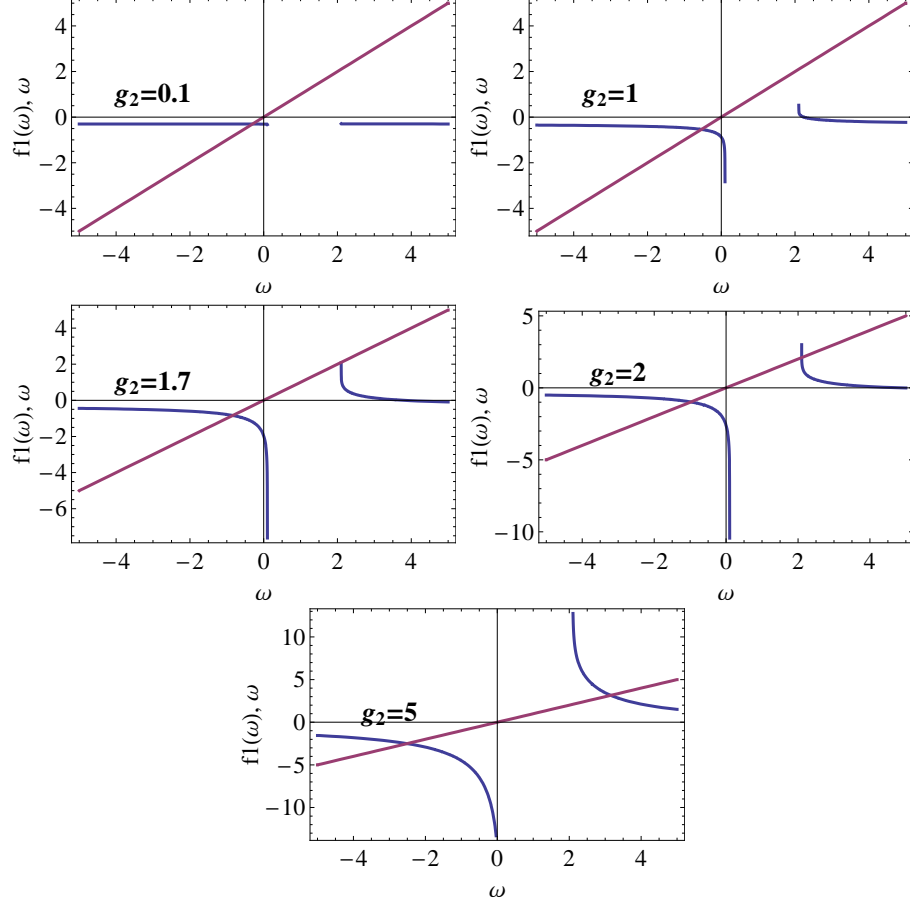


FIG. 2: variation of  $f1(\omega)$  with  $\omega$  and solutions for  $f1(\omega) = \omega$  for different values of  $g_2$

Thus, the fluctuation in  $\phi$  can undergo oscillation with 1) one frequency 2) two frequencies 3) no frequency at all, depending on the value of  $g_2$ .

In Case 1,  $\tilde{\phi}(t) = \phi_1 e^{i\omega_1 t}$ , or,  $\phi(t) = \phi_0 + \phi_1 e^{i\omega_1 t}$

In Case 2,  $\tilde{\phi}(t) = \phi_1 e^{i\omega_1 t} + \phi_2 e^{i\omega_2 t}$ , or,  $\phi(t) = \phi_0 + \phi_1 e^{i\omega_1 t} + \phi_2 e^{i\omega_2 t}$

In Case 3,  $\phi = \phi_0$ , because the fluctuation decays exponentially.

#### IV. PROBING BY AN OSCILLATORY DRIVE

Let us add a small oscillatory component to the magnetic field. So, instead of  $g_2$ , the coupling is of the form  $g_2(1 + \epsilon e^{i\Omega t})$  where  $\epsilon \ll 1$ . We can make a perturbative expansion

$$\begin{aligned}\tilde{\phi}(t) &= \phi^0 + \epsilon \phi' \\ \tilde{O}(t) &= O^0 + \epsilon O'\end{aligned}$$



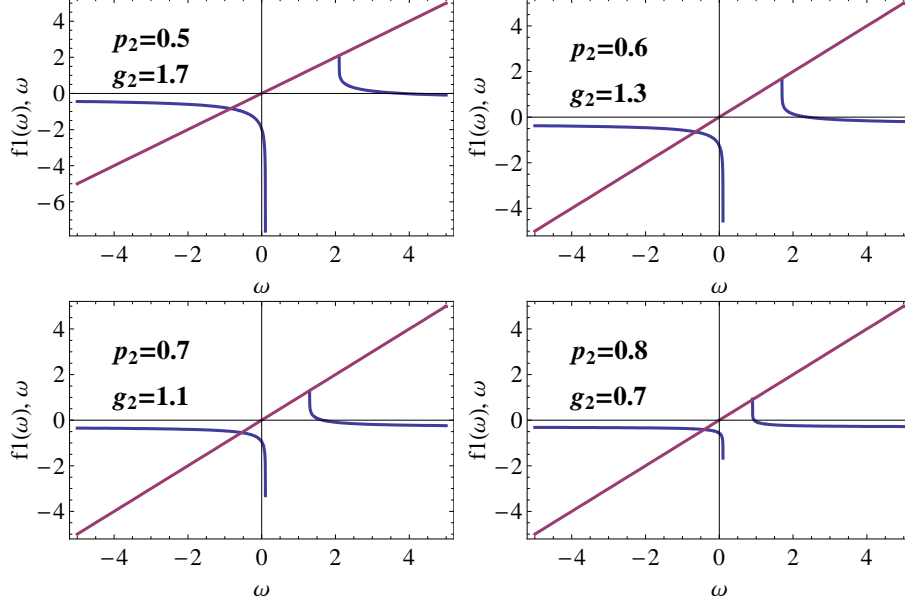


FIG. 3: variation of  $f_1(\omega)$  with  $\omega$  and critical coupling for different values of population imbalance.

Where  $\phi^0$  and  $O^0$  are the values of  $\tilde{\phi}(t)$  and  $\tilde{O}(t)$  when there is no oscillatory part in the coupling. Noting that  $\phi^0 = \phi_1 e^{i\omega t}$ , it follows that

$$i\hbar \frac{\partial O'}{\partial t} = a_p O' + g_2 \phi' + g_2 \phi_1 e^{(\Omega + \omega_1)t} \quad (14)$$

Taking Fourier Transforms as before,

$$\phi'(\omega) = \frac{g_2^2 \phi_1(\omega) 2\pi \delta(\Omega + \omega_1 - \omega) (f(p_1, p_2, \omega_1) + f(p_1, p_2, \omega))}{\epsilon_b + \hbar\omega - g_2^2 f(p, \omega)} \quad (15)$$

Here  $f(p_1, p_2, \omega) = f(p_2, \omega) - f(p_1, \omega)$ . Now,  $\phi'(\omega)$  is non-zero only when  $\omega = \omega_1 + \Omega$  and its value at that particular frequency is

$$\phi'(\omega) = \frac{g_2^2 \phi_1(\omega) 2\pi (f(p_1, p_2, \omega_1 + \Omega) + f(p_1, p_2, \omega_1))}{\epsilon_b + \hbar(\Omega + \omega_1) - g_2^2 f(p_1, p_2, \omega_1 + \Omega)} \quad (16)$$

There is a resonance when the denominator becomes zero, i.e.,  $\epsilon_b + \hbar(\Omega + \omega_1) - g_2^2 f(p_1, p_2, \omega_1 + \Omega) = 0$ . But we know,  $\epsilon_b + \hbar\omega_1 - g_2^2 f(p_1, p_2, \omega_1) = 0$ . Subtracting,

$$\hbar\Omega + g_2^2 (f(p_1, p_2, \omega_1) - f(p_1, p_2, \omega_1 + \Omega)) = 0 \quad (17)$$

Since  $\Omega$  is associated with the Feshbach resonance, by tuning the frequency of the time-dependent magnetic field, we can control  $\Omega$ . If, for a particular  $\Omega$  the above equation is

satisfied, then we have a sharp resonance in the fluctuation is  $\phi$ .

If,  $\phi^0 = \phi_1 e^{i\omega_1 t} + \phi_2 e^{i\omega_2 t}$ , following the same treatment,  $\phi'(\omega)$  is non-zero only when  $\omega = (\omega_1 + \Omega)$  or  $(\omega_2 + \Omega)$ , and its values at those frequencies are

$$\frac{g_2^2 \phi_1(\omega) 2\pi (f(p_1, p_2, \omega_1 + \Omega) + f(p_1, p_2, \omega_1))}{\epsilon_b + \hbar(\Omega + \omega_1) - g_2^2 f(p, \omega_1 + \Omega)} \text{ and } \frac{g_2^2 \phi_2(\omega) 2\pi (f(p, \omega_2 + \Omega) + f(p_1, p_2, \omega_2))}{\epsilon_b + \hbar(\Omega + \omega_2) - g_2^2 f(p_1, p_2, \omega_2 + \Omega)}$$

respectively.

Therefore, resonance will take place when  $\Omega$  satisfies either of the equations:

$$\begin{aligned} \hbar\Omega + g_2^2(f(p_1, p_2, \omega_1) - f(p_1, p_2, \omega_1 + \Omega)) &= 0 \\ \hbar\Omega + g_2^2(f(p_1, p_2, \omega_2) - f(p_1, p_2, \omega_2 + \Omega)) &= 0 \end{aligned} \tag{18}$$

It is obvious that  $\Omega = 0$  is a trivial solution of the equations. Again we take resort to graphical solutions to find out whether non-zero real solutions exist for  $\Omega$ .

Let  $g_2^2(f(p_1, p_2, \omega_1) - f(p_1, p_2, \omega_1 + \Omega)) = f'(\Omega)$

So we plot  $\Omega$  along X-axis and both  $f'(\Omega)$  and  $\Omega$  along Y-axis. The blue lines correspond to  $f'(\Omega)$  and the red lines mark  $\Omega$ . If the two lines cut at any point, we get real solutions of the equation at that point.

Experimentally one can detect the oscillation frequencies. If an oscillatory component is added to the Feshbach coupling, and its frequency is tuned, then at a particular frequency there is a sharp resonance in the condensate fraction. We propose an algorithm for determining the imbalance: chose a  $p_1$  ( $p_2$  is set to 1 because at zero temperature it is the Fermi momentum of the majority species, by which all momenta are scaled) so that the numerically computed oscillation frequencies match with those obtained in the experiment. Then use Those frequencies and the external resonant frequency to calculate  $p_1$ . If this doesn't match with the initial  $p_1$ , use the new values as input for the first step. Iterating the process a number of times, one should arrive at the actual value of the breaching point, or the point which marks the separation in the momentum space.

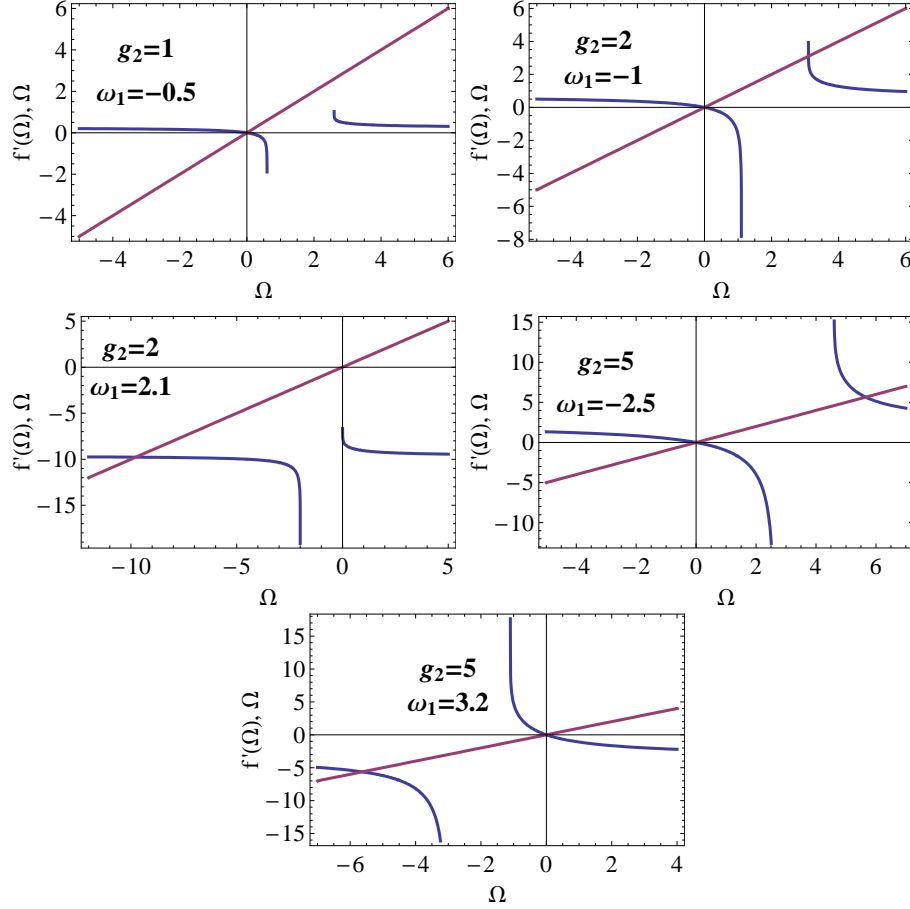


FIG. 4: variation of  $f'(\Omega)$  with  $\Omega$  and solutions for  $f'(\Omega) = \Omega$  for different values of  $g_2$  and corresponding  $\omega$ .

## V. SUMMARY AND DISCUSSION

Here we have studied the natural dynamics of a population-imbalanced fermionic system capable of making a BCS-BEC crossover. We have shown that the condensate fraction shows a periodic or a quasi periodic oscillation, depending on the value of the Feshbach coupling. There is a critical coupling below which the oscillation is always periodic, and its value depends on the population imbalance.

We have shown that if there is an oscillatory component in the Feshbach term, one can achieve a sharp resonance in the condensate fraction by tuning the frequency of the external magnetic field. The breaching momentum can be calculated from this frequency value. Thus it proves to be an indirect method of experimentally determining the momentum space

structure of the imbalanced Fermi system.

This treatment can be extended to detect more complicated structures in the momentum space, too. The FFLO state, which arises due to finite momentum pairing, should also get reflected in a similar dynamical study. The finite pairing momentum is then embedded in the Bosonic field. If we use a single wave number  $q$  in operator  $\phi$  in our original Hamiltonian to represent the finite momentum, the oscillatory dynamics and the resonant frequencies should contain information regarding  $q$ , and thus portray both the momentum structure and the nature of pairing.

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